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Effective Thermal Conductivity of a Saturated Porous Medium

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Nomenclature

k = thermal conductivity, W/m K
Q' = heating rate per unit length of wire, W/m
T = temperature, °C
t = time, s
α' = dimensionless heat capacity to normalize the properties of the heated wire
λ = ratio of solid–fluid conductivities k_s/k_f
τ = Fourier's number

Subscripts
eff = effective
f = fluid
s = solid
w = evaluated at the wire
0 = initial condition

Introduction

POROUS material such as sand, crushed rocks, or gravel, with the influence of local pressure difference, migrates and transfers heat energy. This phenomenon can be encountered in the petroleum and geothermal industries. Convection currents in mineral fluids embedded in the Earth crust or convection of heavy fluids in the Earth mantle next to the Earth crust are examples of practical situation in the petroleum industries. In this situation the transport phenomena can be modeled as porous medium and the effective thermal conductivity must be measured in order to correctly predict the convective characteristics of the different grades of the fluids. Heat and mass transfer in porous media also have a wide range of applications in energy storage systems, insulation materials, cores of nuclear reactors, soils, and material handling processes, such as resin transfer molding (RTM) and structural reaction injection molding (SRIM).

A few experimental methods to determine the effective thermal conductivity has been reported in the literature. A review of theoretical models for calculating the effective thermal conductivity of porous medium is provided by Kaviany.[1]

In this investigation, a nonintrusive transient hot wire technique is used to measure the effective thermal conductivity of saturated porous media. We measured the effective thermal conductivity of a porous material saturated in four different fluids and studied the effects of the thermal conductivity of the fluid on the resulting effective conductivity of the porous matrix. Glass beads of 1.0 and 5.0 mm diameter were utilized. Various fluids, such as air, water, engine oil (SAE 30), and ethylene glycol (commercial antifreeze) were used as the saturating fluid.

Experimental Method and Theory

The theory of heat conduction from a line source in an infinite medium will be employed. For large values of τ, and neglecting the contact resistance between the wire and medium, the temperature of the wire is given by Carlslaw and Jaeger[2] as

$$T_w = \frac{Q'}{(4\pi k)}\left\{\ln\left(\frac{4\tau}{1.7811}\right) + \left[\frac{(\alpha' - 2)}{2\tau\alpha'}\ln\left(\frac{4\tau}{1.7811}\right)\right] + \cdots\right\} \tag{1}$$

This applies directly to the needle probe method, but for this investigation, the heated wire will be mounted on an insulating block. The wire and block are placed against the medium being investigated. Perfect insulator assumption means the heat transfer from the wire can only be through the uninsulated-half of its surface. This will cause the wire temperature to rise twice as fast as in the infinite medium case. Thus, the model described by Eq. (1) must be corrected for this experimental approach by multiplying the actual Q' by a factor of 2. For large values of τ that will occur with the passage of time, the model predicts the temperature of the wire will approach an asymptote and increase linearly as a function of the $\ln(t)$, as follows:

$$T_w = [Q'/(2\pi k)]\ln(4\tau/1.7811) \tag{2}$$

It follows that the effective thermal conductivity of the saturated porous medium can be calculated from the slope of this line by

$$k = [Q'/2\pi(T - T_0)]\ln(t/t_0) \tag{3}$$

The experiments were performed using the apparatus shown in Fig. 1. The probe in this investigation consisted of an insulating mounting foam block (2.5 cm thick, 7 cm wide, and 10 cm long), heated nichrome wire, and thermocouple. The nichrome wire was mounted on one face of the polystyrene foam block to provide a 5-cm-long line source of heat. The

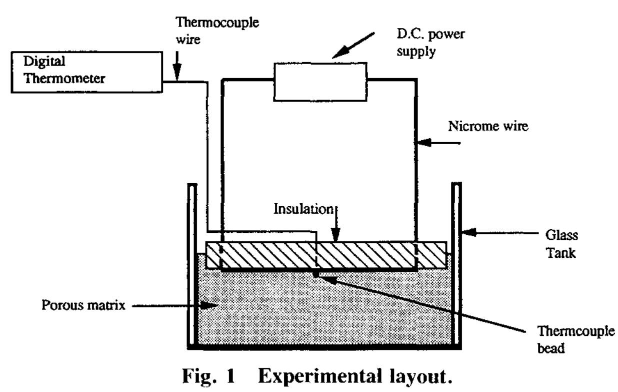

Fig. 1 Experimental layout.

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Table 1 Measured and calculated effective thermal conductivity

Saturating fluid	$\lambda = k_s/k_f$	k_{eff}, 1-mm bead, W/m K Measured	Ref. 1	Ref. 7	Ref. 5	k_{eff}, 5-mm bead, W/m K Measured	Ref. 1	Ref. 7	Ref. 5
Air	39.54	0.23	0.665	0.137	0.208	0.192	0.635	0.125	0.176
Water	1.697	0.828	0.882	0.861	0.862	0.965	0.869	0.847	0.836
SAE 30 oil	7.172	0.570	0.709	0.465	0.493	0.432	0.682	0.440	0.464
Glycol	4.127	0.665	0.748	0.610	0.624	0.647	0.725	0.586	0.597

ends of the wire were passed through the block and was then connected to the dc power supply. The total length of the wire is 1.49 m long with a resistance of 5.5 Ω. A thermocouple was inserted through the block such that the bead was firmly in contact with the nichrome wire and they were all homogeneous with the porous matrix. The nichrome wire and thermocouples were secured to the foam with a thin layer of epoxy. No epoxy was applied between the thermocouple bead and the nichrome wire. In addition, the bottom and four sides of the insulating block were coated with epoxy to prevent absorption of the fluids into the foam. Effects of natural convection in the fluid were minimized by restricting the heating rate. Being a transient method, data are taken before thermal degradation becomes significant. The porous matrix consisted of various combinations of spherical soda-lime glass beads and saturating fluids (Table 1), arranged in a bed approximately 7–10 cm deep in a 35-l glass tank. If the k_s is large, heat conduction at the point of contact becomes very important in determining k_{eff}.

For each bead size/fluid combination, constant voltages of 4, 6, 8, and 10 V from a Hewlett Packard dc power device were applied to the wire for a limited period of time. Nichrome wire has a low-temperature coefficient of resistivity, and thus, the heat generation can be assumed constant along the length of the wire. Wire temperature was recorded every 10 s with an Omega 871 Digital Thermometer.

To compare the conductivity of the insulator with that of the porous layer, and allow comparison of the data with data reported in the literature, the thermal conductivity of the insulation block was measured and has been reported elsewhere.[3] The ratio of the conductivity of the insulator to the effective conductivity of the medium for all the fluid/bead combinations is such that the assumption of the symmetry of isotherm in the test medium is justified, and the assumption that the mounting block is a perfect insulator is valid. The conductivity of the insulator is less than 5% of the effective conductivity of the water-saturated medium, thus, any heat loss from the the thin wire through the insulation will be small compared to the heat flow into the porous matrix. Heat loss for the air-saturated medium is much higher (20%), which may result in an overestimation of k_{eff}.

The porosity of the glass bead bed was determined by filling a graduated 250-ml beaker to the 100 ml gradation with beads, weighing the filled beaker, adding water to the 100 ml level, and weighing the beaker again. The packing of the beads was random, and no compaction of the beads was performed. The difference in weight allows the volume of water (the porous volume) to be determined. Wall effect, especially for the 5-mm bead, may cause the measured porosity from the 100-ml beaker to be slightly different from that in the test tank. The porosity was determined to be 40% for the 5.0-mm-diam beads and 37% for the 1.0-mm-diam beads.

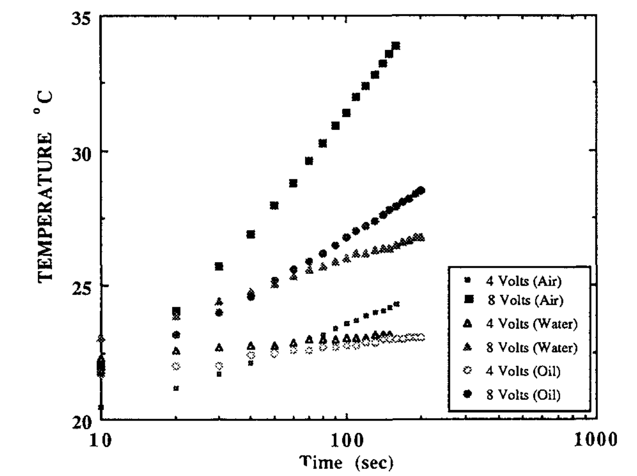

Fig. 2 Typical temperature response of wire in the saturated porous medium.

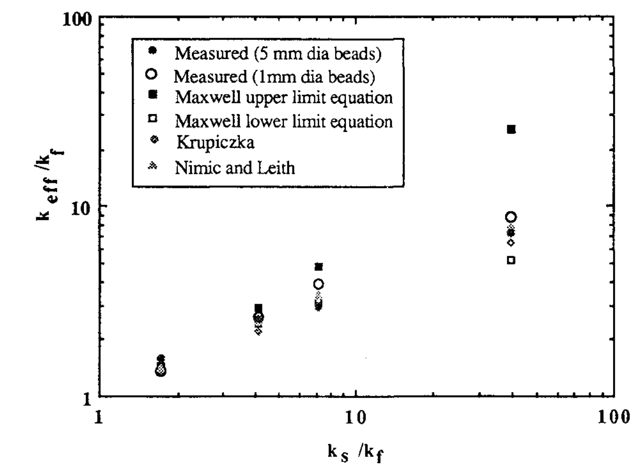

Fig. 3 Experimental and theoretical values of effective thermal conductivity.

Error Analysis

Prior to the start of the experiment, the thermocouple was verified to be calibrated within 0.04°C. The power supply was accurate to within 0.01% for any given constant voltage. Uncertainty from the thermophysical properties were not included in the error analysis. Large bead size appears to introduce uncertainty in the measurements, possibly due to packing effects at the surface of the probe and the fact that natural convection can be more easily induced with the larger void spaces that are present with the large beads. The individual measurement uncertainties used in the error analysis include voltage, current, wire length, and the slope of the temperature response. Using the fractional overall error as the square root of the sum of the squares of the errors of each contributor, the experimental uncertainty was calculated to be in the range of 4.1–16%. Details of the error analysis are given in Ref. 3.

Results

Figure 2 shows an example of the temperature data obtained over time. The data is consistent with the theory, with a very clearly defined asymptotic region. Some data obtained for the 5.0-mm beads with 10 V applied showed a reduction in slope, which is probably due to the effects of natural convection and the measurement uncertainties.

The conductivity of the insulating block was determined to be 0.04 W/m K, which is in good agreement with published data. The measured effective thermal conductivity together with prediction made by some theoretical models are presented in Table 1. One such model is the mixing rule based on volume fraction and reported in Imadojemu and Porter.[4] The measured k_{eff} agrees with the predicted value if $\lambda \cong 1$ and the model overestimates the measured k_{eff} for other values of λ. The difference may be due to the presence of natural convection, especially for the air medium and the large size of the beads. Nimick and Leith[5] have developed an equation to empirically address the case where interaction of the spheres could not be neglected and for which the porosity vary. It is well-known that variation in porosity and wall-channeling effect can have some effects on k_{eff}. There seems to be a better agreement with the measured k_{eff} for all values of λ reported when compared with Nimick and Leith. Their equation adequately accounts for the variations in the fluid thermal conductivity when the solid material is granular, silica-based, and the thermal conductivity is less than the solid material's conductivity. Measured k_{eff} for the glass/water matrix is also in agreement with the results of Tanaka and Miyazawa.[6]

In Fig. 3 k_{eff} is presented in a dimensionless form. For $\lambda \cong 1$ there is good agreement between the measured and predicted values of k_{eff} from all the models. It is clear that the measured k_{eff} is dependent on λ. The measured data are also compared to the analytical solution of Maxwell.[7] The upper bound equation represents low conductivity liquids in a high conductivity solid matrix, whereas the lower bound equation represents solid spheres suspended in a fluid matrix. Maxwell's equations are only applicable to granular porous media with porosities of 0.3–0.5. Krupiczka[8] correlation based on a two-dimensional conduction equation seems to agree well with Maxwell's lower bound equation, while Nimick and Leith[5] is seen to predict our measured k_{eff} fairly well, even for the air and large bead size combination, thus supporting the general applicability of that model. Figure 3 shows the data obtained in this investigation is consistent with the experimental data and conclusion of Prasad et al.[9] and other emperical models in the literature.

Conclusions

The method employed in this study is accurate and can be employed in situ. The effective thermal conductivity of a porous medium depends on the microstructure of the solid, porosity, and permeability of the matrix, the conductivities of the two constituents, and slightly on geometrical consideration. In this investigation, however, there was no strong dependence of the effective thermal conductivity on the bead size, but rather the dependence was on the saturating medium. The transport property that plays a major role in the effective conductivity of the porous matrix seems to be the fluid's thermal diffusivity and conductivity.

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